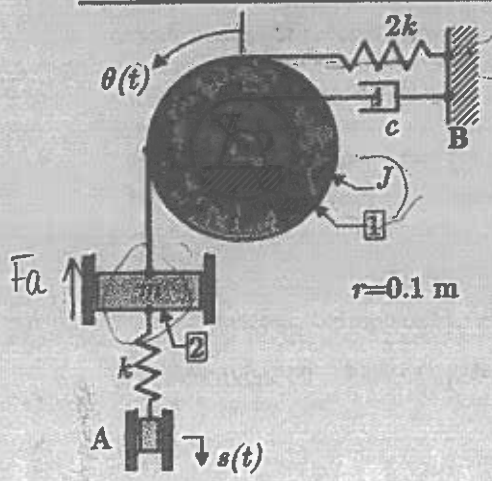


2) (6 val.) O sistema representado na figura é constituído por:

- corpo 1, polia que roda em torno do ponto **O**, com momento de inércia J, e que está ligada ao exterior através de uma mola linear elástica de rigidez $2k$ e de um amortecedor viscoso de constante c;
- corpo 2, massa m que translada verticalmente e que está ligada à polia através de um tirante e a uma mola linear elástica de rigidez k que, na outra extremidade, está submetida a um deslocamento $s(t)$.

Na situação de equilíbrio estático, $\theta(t) = 0$ rad e $s(t) = 0$ m.

a) (20%) Estabelecer a equação diferencial (linearizada) do movimento.

Para resolver as alíneas seguintes, assumir que a equação do movimento é a seguinte:

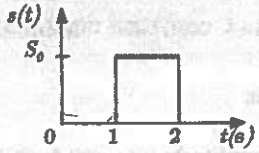
$$0.1\ddot{\theta}(t) + 0.1\dot{\theta}(t) + 192\theta(t) = 320s(t)$$

b) (20%) Determinar a(s) frequência(s) do movimento harmónico $s(t) = 0.02\cos(\omega t)$ para a qual(s) a amplitude de deslocamento da massa, em regime estacionário, não ultrapassa 10 mm.

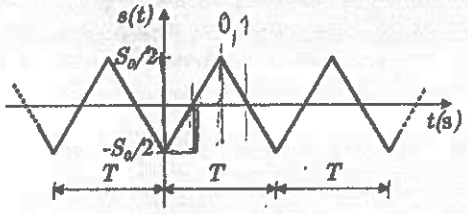
c) (20%) Nas condições da alínea anterior, determinar a amplitude máxima da componente horizontal da força no apoio **O**.

Na resolução das alíneas seguintes, assumir que o amortecimento é nulo.

d) (20%) Determinar uma expressão para a rotação do disco no intervalo $1 \le t \le 2$ s, quando ao disco é aplicada a perturbação inicial ($t = 0$ s) $\theta_0 = 0$ rad e $\dot{\theta}_0 = 1$ rad/s e o ponto **A** é sujeito ao deslocamento transiente $s(t)$ representado na figura seguinte.



e) (20%) Admitindo que o deslocamento $s(t)$ é periódico com período $T = 0.2$ s e com a forma representada na figura seguinte, obter uma estimativa para o deslocamento angular do disco (pretende-se a melhor estimativa dada por uma expressão com apenas um termo função do tempo).



$\frac{S_0/2 \cdot t}{T/4}$

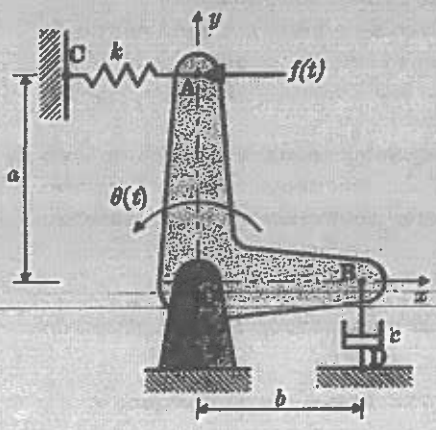
Nota:

$$\int x\sin(ax)\mathrm{d}x = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} \qquad \int x\cos(ax)\mathrm{d}x = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a}$$

$$\int x^2\sin(ax)\mathrm{d}x = \frac{1}{a^3}\left(-a^2x^2\cos(ax) + 2\cos(ax) + 2ax\sin(ax)\right)$$

$$\int x^2\cos(ax)\mathrm{d}x = \frac{1}{a^3}\left(a^2x^2\sin(ax) - 2\sin(ax) + 2ax\cos(ax)\right)$$

FEUP, 29.04.2009
José Dias Rodrigues
Pedro Leal Ribeiro



1) (6 val.) A barra em forma de **L** representada na figura oscila em torno do ponto **O**. A barra está ligada ao exterior, através de uma mola linear de rigidez k no ponto **A** e através de um amortecedor viscoso de coeficiente c no ponto **B**. O momento de inércia da barra em relação a um eixo que passa por **O**, perpendicular ao plano Oxy, é J. No ponto **A** está aplicada uma solicitação $f(t)$ na direcção horizontal.

Considerando que na posição de equilíbrio estático o eixo **OA** está na vertical, determinar:

a) (20%) A equação diferencial (linearizada) de movimento do sistema.

Para a aplicação numérica,

$k = 1200$ N/m $\quad c = 40$ Ns/m $\quad J = 0.1$ kgm^2 $\quad a = 0.15$ m $\quad b = 0.12$ m

determinar:

b) (10%) A razão de amortecimento;

c) (10%) A frequência angular amortecida e o respectivo período.

Para uma perturbação inicial $\theta_0 = 10°$ e $\dot{\theta}_0 = 0$ rad/s, determinar,

d) (10%) o número de ciclos necessários para que a amplitude se atenue para um valor inferior a 5% de θ_0;

e) (15%) a energia dissipada até ao instante em que o sistema passa pela segunda vez na posição de equilíbrio estático.

Para uma solicitação harmónica $f(t) = F\cos\omega t$ com uma amplitude de 50 N e uma frequência de 3 Hz, determinar:

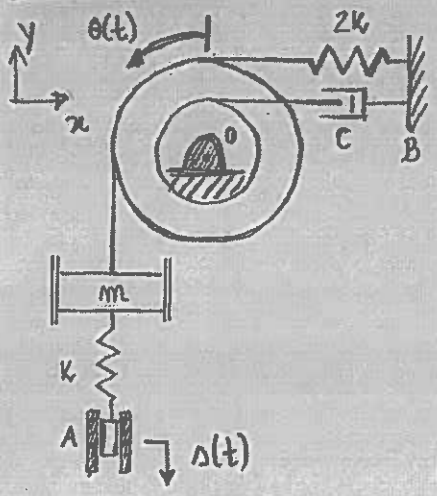
f) (15%) a resposta estacionária do sistema;

g) (10%) a força transmitida ao apoio **D**;

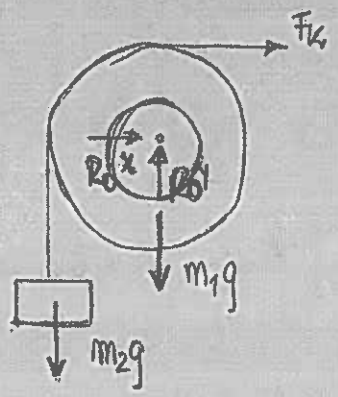
h) (10%) a frequência de ressonância e a amplitude máxima de resposta no ponto **A**.

1ª prova de avaliação 2008/2009

PROBLEMA (2)

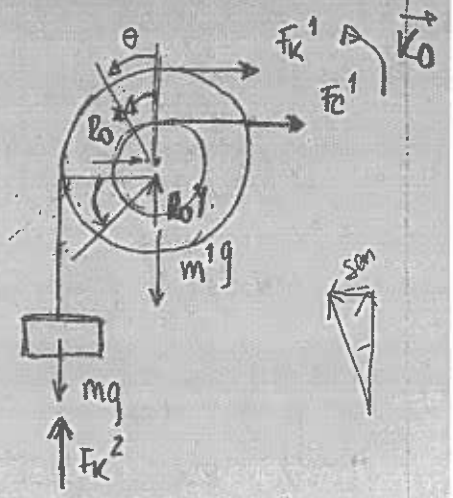


EQUILIBRIO ESTÁTICO



$$2r \cdot F_{k_1} = 2r\, m_2 g$$
$$k\delta_0 = m_2 g$$

EQUILIBRIO DINÂMICO



$$-K\Delta\ell = -K\left(-2r\theta - (-s(t)\right)$$

a) $\Sigma \vec{M}_0 = \vec{K}_0$

$$\begin{vmatrix} -2r\,\mathrm{sen}\theta \\ 2r\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 2k2r\,\mathrm{sen}\theta - ks \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -r\,\mathrm{sen}\theta \\ r\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} cr\dot{\theta}\,\mathrm{sen}\theta \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -2r\cos\theta \\ -2r\,\mathrm{sen}\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ -m_2 g \\ 0 \end{vmatrix} +$$

$$\begin{vmatrix} -2r\cos\theta \\ -2r\,\mathrm{sen}\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ k\cdot 2r\,\mathrm{sen}\theta - ks(t) \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ J_0\ddot{\theta} \end{vmatrix}$$

$$J_0 = J + \underbrace{J_2}_{=0} + m(2r)^2 = J + 4mr^2$$

$$-8r^2k\theta - \cancel{m_2 g}\cdot 2r - cr^2\dot{\theta} + 2r\cancel{m_2 g} - 2r\left(k2r\theta - ks(t)\right) = (J + 4mr^2)\ddot{\theta}$$

$$(J+4mr^2)\ddot{\theta} + cr^2\dot{\theta} + \left(k\cdot 8r^2 + 4r^2k\right)\theta = 2rks(t)$$

$$\underbrace{(J+4mr^2)}_{m_{eq}}\ddot{\theta} + \underbrace{cr^2}_{c_{eq}}\dot{\theta} + \underbrace{12kr^2}_{k_{eq}}\theta = \underbrace{2rk}_{f_{eq}}s(t)$$

$$\underbrace{0,1}_{meq}\ddot{\theta}(t) + \underbrace{0,1}_{ceq}\dot{\theta}(t) + \underbrace{192}_{keq}\theta(t) = \underline{320}\,s(t)$$

b) $s(t) = 0,02\cos(\omega t) \longrightarrow X(\omega) \leq 10\,mm$

$\omega = ?$

$$x(t) = X(\omega)\cos(\omega t - \phi)$$

$$\omega_n = \sqrt{\frac{keq}{meq}} = \sqrt{\frac{192}{0,1}} = 43,82\ rad/s$$

$$\xi = \frac{ceq}{2\,meq\,\omega_n} = \frac{0,1}{2\times 0,1\times 43,82} = 0,011$$

$$\theta(\omega) = \frac{\theta_s}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}} = \frac{\frac{feq}{keq}}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}$$

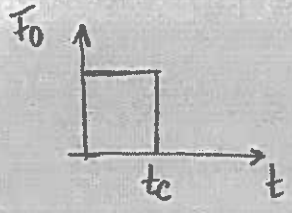
com $feq = 320 \times 0,02 = 6,4$

$$0,01 = \frac{\frac{6,4}{192}}{\sqrt{(1-\beta^2)^2+(2\times 0,011\times\beta)^2}} \quad \therefore \beta = \begin{cases} -2,08 \times \\ 2,08 \end{cases}$$

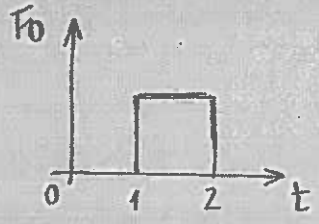
$$\beta = \frac{\omega}{\omega_n} \quad \therefore \quad \omega = 2,08 \times 43,82 = 91,14\ rad/s$$

c) $$\theta(\omega) = \frac{feq/keq}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}} =$$

d) Para $t = 0s$: $\theta_0 = 0$ rad e $\dot{\theta}_0 = 1$ rad/s



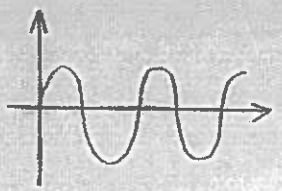
$$0 \leq t \leq t_c \therefore x(t) = \frac{F_0}{K_v}(1 - \cos \omega_m t)$$



$$1 \leq t \leq 2s$$

$$x(t-1) = \frac{F_0}{K_v}(1 - \cos \omega_m (t-1))$$

+



$$x(t) = A \cos(\omega_m t - \phi)$$

Assim para $1 \leq t \leq 2s$:

$$\theta(t) = A \cos \omega_m t + \frac{F_0}{K_v}(1 - \cos \omega_m (t-1))$$

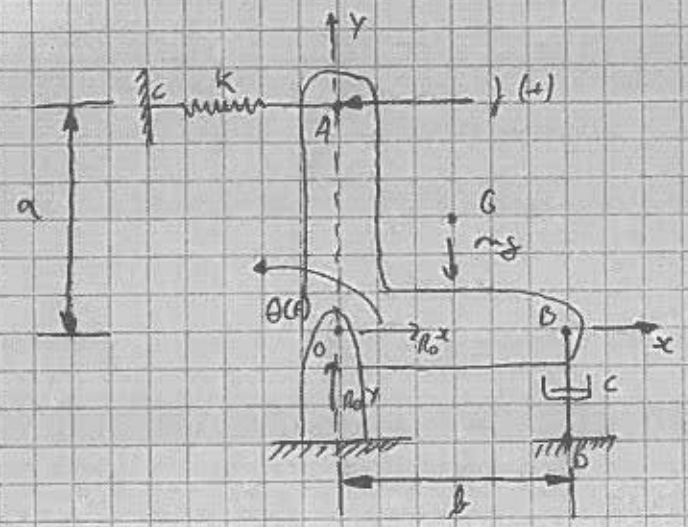
e) Regime periódico com $T = 0{,}2\,s$ $\left(\omega = \frac{2\pi}{T} = \frac{2\pi}{0{,}2} = 31{,}4 \text{ rad/s}\right)$

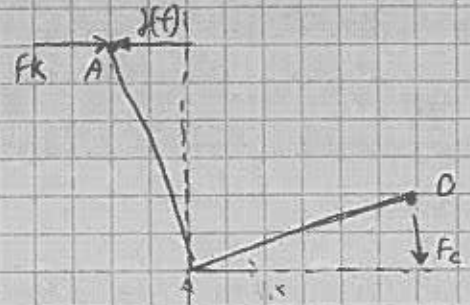
$\beta = \frac{\omega}{\omega_m}$

$\omega = 91{,}14$ rad/s $\rightarrow$ $\omega = 2{,}08$ rad/s $\Rightarrow$ 3º harmônico ($p=3$)

$$s(t) = C_0 + \sum_{p=3}^{\infty} C_p \cos(p\omega t - \psi_p)$$

$$s(t) = \begin{cases} \end{cases}$$





$\vec{F} = \vec{0} \qquad R_{0y} = mg$

$\vec{M} = \vec{K} \Rightarrow \vec{OA} \times \vec{FK} + \vec{OA} \times \vec{f(t)} + \vec{OB} \times \vec{Fc} = \vec{K_0}$

$$\vec{OA} = \begin{vmatrix} -a\sin\theta \\ a\cos\theta \\ 0 \end{vmatrix} \quad \vec{OB} = \begin{vmatrix} b\cos\theta \\ b\sin\theta \\ 0 \end{vmatrix} \quad \vec{FK} = -Ks = \begin{vmatrix} Ka\sin\theta \\ 0 \\ 0 \end{vmatrix} \quad \vec{Fc} = -c\dot{x} = \begin{vmatrix} 0 \\ -Cb\dot{\theta}\cos\theta \\ 0 \end{vmatrix}$$

$$\vec{K_0} = \vec{H_0} = \begin{vmatrix} 0 \\ 0 \\ J\ddot{\theta} \end{vmatrix}$$

$$\begin{vmatrix} -a\sin\theta \\ a\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} Ka\sin\theta \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -a\sin\theta \\ a\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} -f(t) \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} b\cos\theta \\ b\sin\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ -Cb\dot{\theta}\cos\theta \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ J\ddot{\theta} \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ 0 \\ -Ka^2\cos\theta\sin\theta \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ af(t)\cos\theta \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ -Cb^2\dot{\theta}\cos^2\theta \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ J\ddot{\theta} \end{vmatrix}$$

$$-Ka^2\cos\theta\sin\theta + af(t)\cos\theta - Cb^2\dot{\theta}\cos^2\theta = J\ddot{\theta}$$

linealizando $\cos\theta \approx 1 \quad \sin\theta \approx \theta$

$$-Ka^2\theta + af(t) - Cb^2\dot{\theta} = J\ddot{\theta} \Rightarrow J\ddot{\theta} + Cb^2\dot{\theta} + Ka^2\theta = af(t)$$

$m_{eq} = J$
$C_{eq} = Cb^2$
$K_{eq} = Ka^2$
$f_{eq} = af(t)$

$C = 40\ Ns/m$
$K = 1200\ N/m$
$J = 0,1\ kgm^2$
$a = 0,15$
$b = 0,12$

a)

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{Ka^2}{J}} = \sqrt{\frac{1200 \times 0,15^2}{0,1}} = 270\ rad/s$$

$$\zeta = \frac{C_{eq}}{2m_{eq}\omega_n} = \frac{Cb^2}{2J\omega_n} = \frac{40 \times 0,12^2}{2 \times 0,1 \times 270} = 0,0107$$

c)

$$\omega_d = \omega_n \sqrt{1-\xi^2} = 270 \times \sqrt{1-0{,}0107^2} = 269{,}985 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{269{,}985} = 0{,}0233 \text{ s}$$

d) $\theta_0 = 10° = \frac{2\pi \times 10}{360} = 0{,}1745$ rad

$\dot{\theta}_0 = 0$ rad/s

$N = ?$

$\delta = 0{,}05 \Rightarrow \delta N = \xi \omega_n T N$

5% $\theta_0 = 0{,}05 \times 0{,}1745 = 0{,}00875$

$$\ln \frac{0{,}1745}{0{,}00875} = \xi \omega_n T N \qquad N = \frac{1}{0{,}0107 \times 270 \times 0{,}0233} \times 2{,}9929$$

$N = 44{,}46 \simeq 45$ cycles

e) $\Delta E = E_0 - E_t$

$$t|_{x(t)=0} = \frac{1}{\omega_d}\left(\frac{2n-1}{2}\pi + \phi\right) = \frac{1}{269{,}985}\left(\frac{3}{2}\pi + 0{,}0107\right) = 0{,}0175 \text{ s}$$

$$\phi = \tan^{-1}\left(\frac{\dot{x}_0 + \xi\omega_n x_0}{\omega_n\sqrt{1-\xi^2}\, x_0}\right) = \tan^{-1}\left(\frac{0 + 0{,}0107 \times 270 \times 0{,}1745}{270\sqrt{1-0{,}0107^2}\ 0{,}1745}\right) = 0{,}0107$$

$$E_0 = \frac{1}{2} m_{eq} \cancel{\dot{\theta}^2}^{\,=0} + \frac{1}{2} K_{eq} \theta^2 = \frac{1}{2} K a^2 \theta^2 = \frac{1}{2} 1200 \times 0{,}15^2 \times 0{,}1745^2$$

$E_0 = 0{,}411$ J

$$A = \sqrt{\left(\frac{\dot{x}_0 + \xi\omega_n x_0}{\omega_n\sqrt{1-\xi^2}}\right)^2 + x_0^2} = \sqrt{\left(\frac{0 + 0{,}0107 \times 270 \times 0{,}1745}{270\sqrt{1-0{,}0107^2}}\right)^2 + 0{,}1745^2}$$

$= 0{,}17451$

$$\dot{\theta}(t) = A e^{-\xi\omega_n t}\left(\omega_n \cos(\omega_d t - \phi - \psi)\right)$$

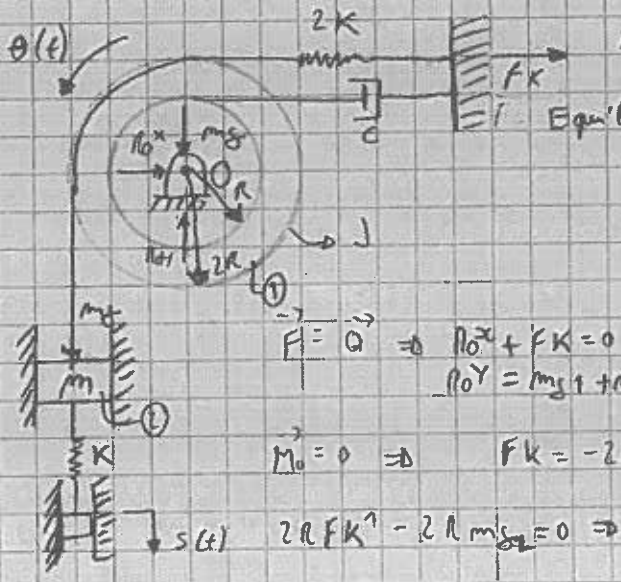
$$\psi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \frac{\sqrt{1-0{,}0107^2}}{0{,}0107} = 1{,}5601$$

$$\dot{\theta}(t) = 0{,}17451\, e^{-0{,}0107 \times 270 \times 0{,}0175}\left(270 \cos(269{,}985 \times 0{,}0175 - 0{,}0107 - 1{,}5601)\right)$$

$\dot{\theta}(t) = -28{,}42$

$$T = \frac{1}{2} \times 0{,}1 \times (-28{,}42)^2 = 40{,}31$$

2)



$n = 0, 1$

Equilibrio $\theta(t) = 0$ rad
$\Delta(t) = 0$ m

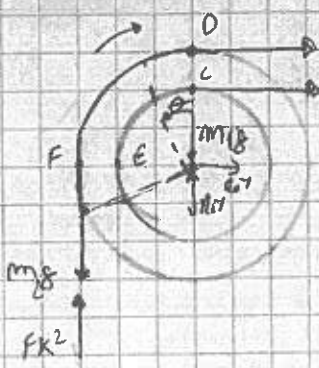
$\vec{F} = \vec{0} \Rightarrow R_0^x + FK = 0 \Rightarrow R_0^x = -FK$
$R_0^Y = mg_1 + mg_2$

$\vec{M_0} = 0 \Rightarrow FK = -2K\delta$

$2RFK^1 - 2R\,mg_2 = 0 \Rightarrow 2RFK = 2R\,mg_2$

$FK^1 = mg$ $\quad R_0Y = FK + mg_2$ $\quad -2K\delta = mg$

$K = \frac{mg}{2\delta}$



$\vec{OC} \times \vec{Fc} + \vec{OD} \times \vec{FK^1} + \vec{OF} \times \vec{mg} + \vec{OF} \times FK^2$

$$\vec{OC} = \begin{vmatrix} -n\sin\theta \\ n\cos\theta \\ 0 \end{vmatrix} \quad \vec{OD} = \begin{vmatrix} -2n\sin\theta \\ 2n\cos\theta \\ 0 \end{vmatrix} \quad \vec{OF} = \begin{vmatrix} -2R\cos\theta \\ -2R\sin\theta \\ 0 \end{vmatrix} \quad \vec{FK_1} = -2K\delta = \begin{vmatrix} +4KR\sin\theta - 2K\delta \\ 0 \\ 0 \end{vmatrix}$$

$$\vec{Fc_1} = -C\dot{x} = \begin{vmatrix} +Cn\dot{\theta}\cos\theta \\ 0 \\ 0 \end{vmatrix} \quad \vec{FK^2} = -K\Delta Y = \begin{vmatrix} -K(-2R\sin\theta + \Delta(t)) \end{vmatrix}$$

$$\begin{vmatrix} -n\sin\theta \\ n\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} Cn\dot{\theta}\cos\theta \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -2R\sin\theta \\ 2R\cos\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 4KR\sin\theta - 2K\delta \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -2R\cos\theta \\ -2R\sin\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ -m_2g \\ 0 \end{vmatrix} +$$

$$+ \begin{vmatrix} -2R\cos\theta \\ -2R\sin\theta \\ 0 \end{vmatrix} \times \begin{vmatrix} 0 \\ -K(-2R\sin\theta + \Delta(t)) \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ J\ddot{\theta} \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ 0 \\ -Cn^2\dot{\theta}\cos^2\theta \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ -8K\pi^2R^2\sin\theta\cos\theta + 4RK\delta\cos\theta + 2Rm_2g\cos\theta - 4R^2K\cos\theta\sin\theta + 2RK\Delta(t)\cos\theta \end{vmatrix} = J\ddot{\theta}$$

Linearizzando $\cos\theta \simeq 1$ e $\sin\theta \simeq 0$

$$J\ddot{\theta} + CR^2\dot{\theta} + 8KR^2\theta - 2Rm_2g\cos\theta + 2Rm_2\cancel{\sin\theta} + 4R^2K\theta = 2RK\Delta(t)$$

$J = J_1 + J_2 = J_{01} + \cancel{J_{02}}^{=0} + m_2 d_2^2 = J_0 + m(2R)^2 = J_0 + 4mR^2$

$(J + 4mR^2)\ddot{\theta} + CR^2\dot{\theta} + 12kR^2\theta = 2R\,k\,\Delta(t)$

$m_{eq} = (J + 4mR^2)$
$c_{eq} = CR^2$
$K_{eq} = 12kR^2$
$f(t)_{eq} = 2Rk$

b) $0{,}1\,\ddot{\theta}(t) + 0{,}1\,\dot{\theta}(t) + 192\,\theta(t) = 320\,\Delta(t)$

$m_{eq} = 0{,}1$
$c_{eq} = 0{,}1$
$K_{eq} = 192$
$f_{eq} = 320\,\Delta(t) = 320 \times 0{,}02 = 6{,}4$

$X(w) = 0{,}010\ m$

$X_S = \frac{F_{eq}}{K_{eq}} = \frac{6{,}4}{192} = 0{,}0333\ m$

$$x(w) = \frac{X_S}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

$$w_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{192}{0{,}1}} = 43{,}82\ rad/s$$

$$\xi = \frac{w_n c_{eq}}{2K_{eq}} = \frac{43{,}82 \times 0{,}1}{2 \times 192} = 0{,}0114$$

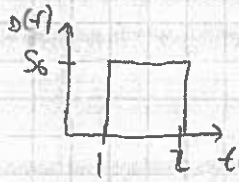
$$0{,}010 = \frac{0{,}0333}{\sqrt{(1-\beta^2)^2 + (2 \times 0{,}0114 \times \beta)^2}} \Rightarrow \beta = 2{,}081$$

$$\beta = \frac{w}{w_n} \Rightarrow w = \beta \times w_n = 43{,}82 \times 2{,}081 = 91{,}19\ rad/s$$

c) $R_0^x = -F_K$

$$F_T = \frac{F\sqrt{1 + (2\xi\beta)^2}}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} = \frac{6{,}4\sqrt{1 + (2 \times 0{,}0114 \times 2{,}081)^2}}{\sqrt{(1-2{,}081^2)^2 + (2 \times 0{,}0114 \times 2{,}081)^2}} = 40{,}55\ N$$

d) $\xi = 0 \qquad t = 0 \Rightarrow \theta_0 = 0\ rad \quad \dot{\theta}_0 = 1\ rad/s$



$\theta(t)$

$$S(t) = \begin{cases} 0 & 0 \le t_c \le 1 \\ S_0 & 1 \le t_c \le 2 \\ 0 & t_c > 2 \end{cases}$$

$$x(t) = \frac{1}{m\,w_d}\int_0^t f(\tau)\, e^{-\xi w_n (t-\tau)} \sin w_d (t-\tau)\, d\tau$$

$$= \frac{1}{m\,w_d}\left(\int_0^1 0\, d\tau + \int_1^2 S_0 \sin w_d (t-\tau)\, d\tau + \int_2^\infty 0\, d\tau\right)$$

$$= \frac{S_0}{m\,w_d}\int_1^2 \sin w_d (t-\tau)\, d\tau = \frac{S_0}{m\,w_d}\int_1^2 \sin w_d t \cos w_d \tau - \cos w_d t \sin w_d \tau\, d\tau$$

$$= \frac{S_0}{m\,w_d}\left(\int_1^2 \sin w_d t \cos w_d \tau - \int_1^2 \cos w_d t \sin w_d \tau\right)$$

$$= \frac{S_0}{m\,w_d}\left[+\sin w_d t\left(-w_d \sin w_d \tau\right) - \cos w_d t\left(w_d \cos w_d \tau\right)\right]_1^2$$

$$= \frac{-S_0}{m\,\cancel{w_d}}\left[+\cancel{w_d} \sin w_d t \sin w_d \tau + \cancel{w_d} \cos w_d t \cos w_d \tau\right]_1^2$$

$$= -\frac{S_0}{m}\left[+\sin w_d t \sin 2 w_d + \cos w_d t \cos 2 w_d - \sin w_d t \sin w_d - \cos w_d t \cos w_d\right]$$

$$= -\frac{S_0}{m}\left(\cos w_d (t+2) - \cos w_d (t+1)\right)$$

e) $\Delta(t)$ $\quad T = 0{,}2\,s$

$\theta(t)$

$$f(t) = \frac{F_0}{2} + \sum_{p=1}^{\infty} A_p \cos(p w t) + \sum_{p=1}^{\infty} B_p \sin(p w t)$$

$$F_0 = \frac{2}{T}\int_0^T f(t)\, dt = \frac{2}{0{,}2}\left[\int_0^{\frac{T}{2}} \frac{2 S_0 t}{T} - \frac{S_0}{2}\, dt + \int_{\frac{T}{2}}^{T} -\frac{3}{2} S_0 + S_0\, dt\right]$$

$$y = m x + b \quad \to m = \frac{\frac{S_0}{1}}{\frac{T}{2}} = \frac{2 S_0}{T} \quad \to f(t) = \frac{2 S_0}{T} t - \frac{S_0}{2} \qquad 0 \le t \le \frac{T}{2}$$

$$m = -\frac{S_0}{2} - \frac{3}{2} S_0 = \frac{-\frac{3}{2} S_0}{\frac{T}{1}} \qquad f(t) = -\frac{3 S_0}{2T} t + S_0 \qquad \frac{T}{2} \le t \le T$$

$$F_0 = \frac{2}{0{,}2}\left[\int_0^{0{,}1} \frac{2 S_0 t}{0{,}2} - \frac{S_0}{2}\, dt + \int_{0{,}1}^{0{,}2} -\frac{3}{2}\frac{S_0 t}{T} + S_0\, dt\right]$$

$$= \frac{2}{0{,}2}\left[\left(\frac{2 S_0 t^2}{0{,}4} - \frac{S_0 t}{2}\right]_0^{0{,}1}\right) + \left(-\frac{3 S_0 t^2}{0{,}8} + S_0 t\right)\Big]_{0{,}1}^{0{,}2}$$

$$= \frac{2}{0{,}2}\left[0{,}05 S_0 - 0{,}05 S_0 + \left(-0{,}15 S_0 + 0{,}2 S_0 - (-0{,}0035 S_0 + 0{,}1 S_0)\right)\right]$$

$$F_0 = \frac{2}{0,2} \times 0,0465\, S_0 = 0,465\, S_0$$

$$A_p = \frac{2}{\pi}\left(\int_0^{0,1}\left(\frac{2S_0 t}{0,2} - \frac{S_0}{2}\right)\cos(p\omega t)\,dt + \int_{0,1}^{0,2}\left(-\frac{3S_0}{2\times 0,2}t + S_0 -\right)\cos(p\omega t)\,dt\right.$$

$$= \frac{2}{0,2}\left(\frac{2S_0}{0,2}\int_0^{0,1} t\cos(p\omega t) - \frac{S_0}{2}\int_0^{0,1}\cos(p\omega t) - \frac{3S_0}{0,4}\int_{0,1}^{0,2} t\cos(p\omega t) + S_0\int_{0,1}^{0,2}\cos p\omega t\right)$$

$$= \frac{2}{0,2}\left(\left[\frac{2S_0}{0,2}\left(\frac{\cos(p\omega t)}{(p\omega)^2} + \frac{t\sin(p\omega t)}{p\omega}\right) - S_0\left(\frac{1}{p\omega}\sin(p\omega t)\right)\right]_0^{0,1} -\right.$$

$$\left.-\left[\frac{3S_0}{0,4}\left(\frac{\cos(p\omega t)}{(p\omega)^2} + \frac{t\sin(p\omega t)}{(p\omega)}\right) + \frac{S_0}{p\omega}\sin(p\omega t)\right]_{0,1}^{0,2}\right)$$

$$= \frac{2}{0,2}\left[\frac{2S_0}{0,2}\left(\frac{\cos(0,1p\omega)}{(p\omega)^2} + \frac{0,1\sin(0,1p\omega)}{p\omega} - S_0\left(\frac{1}{p\omega}\sin(0,1p\omega)\right)\right] - \left(\frac{2S_0}{0,2}\left[\frac{1}{(p\omega)^2} + 0\right]\right.\right.$$

$$-\left[\frac{3S_0}{0,4}\left(\frac{\cos(0,2p\omega)}{(p\omega)^2} + \frac{0,2\sin(0,2p\omega)}{p\omega}\right) + \frac{S_0}{p\omega}\sin(0,2p\omega) - \left(\frac{3S_0}{0,4}\left(\frac{\cos(0,1p\omega)}{(p\omega)^2} + 0,1\right.\right.\right.$$

$$\left.\left.\left.+\frac{0,1\sin(0,1p\omega)}{p\omega}\right) + \frac{S_0}{p\omega}\sin(0,1p\omega)\right)$$

$$= \frac{2}{0,2}$$